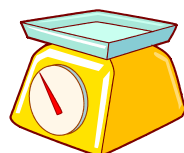


MathsGoGoGo

Mathematics at Home



Tables?
Piece of cake!



x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

This is a free document for you, the parent/guardian. It consists of mathematical ideas for you to try at home, mainly with primary children.

You may distribute the document freely provided you always give the whole document and do not alter it.

www.mathsgogogo.co.uk

Alan Young

Introduction

One day in my first year of teaching, I was sitting having my sandwiches at lunchtime and quite out of the blue my head of department said, “Mr Young...” (long pause while she pointed around the staffroom), “ everywhere I look I see mathematics.”

I was young and foolish at the time and something passed through my head that I could not possibly repeat here (fortunately I didn't say it aloud). However, the strangest thing happened – a few years later I found that everywhere I looked I saw mathematics. As the years pass by this experience just gets stronger and stronger.

I used to use this as an example with my students at both high school and primary school level. I would challenge them to give me an example of something that had no mathematics attached and they would normally say something like “ A window”. Then for the next five minutes I would bore them to death by describing how a window was a rectangle with a length and width and from these we can calculate the area; how the glass had thickness and so the volume could also be calculated; how the window turned on its hinges so rotations were involved; how light, heat and sound passed through the window and so we could calculate the percentage loss of each as it entered the room; how this energy entering the room would raise the temperature of the objects in the room and this increase in temperature could be measured; how glaziers call a piece of glass a ‘square’, even though it is rectangular in shape, thus demonstrating that the same words were sometimes used in different ways by different groups of people; and



It is very easy to find mathematics in just about everything around us and the reason is not surprising – it is simply that mathematics is a language that describes the real world. Early stone age people apparently first used numbers to count their sheep and to perform simple measurements. From this humble beginning we have progressed through the building of Stonehenge and the Pyramids (by people living in the stone and copper ages, incidentally) to the modern engineering we see all around us today. It is only when one studies highly abstract mathematics at university level and above do we break this connection with the ‘real world’ (*mathematicians are a funny lot – they don't normally distinguish between real and imaginary worlds, but that need not concern us here!*).

This, of course, gives us a great way of helping our children with mathematics at home since virtually everything we do, see or touch at home has some mathematics involved. And this is what this document is all about. I am going to give you some of the best ways to inspire and assist your children in not only becoming more proficient, but also in enjoying the subject much more and understanding the relationship between mathematics and our everyday surroundings.

But first I want to give you some general, but very important pointers that you will need to take on board if you are to be successful.

Get into the habit of seeing mathematics everywhere you look. If you don't do this, how can you expect your children to do so? Lead by example. Instead of sitting in front of the television tonight worrying about work tomorrow, focus your mind on one or two objects and tell yourself all the mathematics that you can think of related to those objects as I did with the window earlier. Don't limit yourself to mathematics that your children may be able to understand – really let rip and cover every level *you* are able to handle. If you are really keen you can write it all down in note form.

Remember that **we all learn by making mistakes** – and that means you and it means me and, most of all, it means *your children*. NEVER tell off your children for making mistakes. Instead, discuss the mistake with them and try to understand *why* they made it in order to correct their understanding.

Don't expect your children to understand something just because you think they ought to understand it. I have a friend who is a trained teaching assistant. One day we were discussing how children learn mathematics and she said, "children in our school know that seven eights are fifty six, but they don't know what eight sevens are!" The implication here, of course, is that it is obvious that if seven eights are fifty six, eight sevens must also be fifty six. Well, the truth is that it is not at all obvious to a young child. This is what mathematicians call the 'Commutative Law' and it applies to multiplication ($7 \times 8 = 8 \times 7$) and addition ($6 + 7 = 7 + 6$), but it does not apply to subtraction ($9 - 7 \neq 7 - 9$) or division ($8 \div 2 \neq 2 \div 8$) and children need to learn when this law applies and when it does not (*young children are not normally asked learn the term 'Commutative Law'*).

Use as much mathematical language as possible. It's a funny thing that we enjoy teaching children new words such as 'excavator', 'alsatian' and 'December', but many of us shy away from mathematical words such as 'pentagon', 'bisect' and 'rotation'. This is a shame as mathematics has a great vocabulary, much of it going back to the Ancient Greeks some two thousand years ago.

Once upon a time there was a quadrilateral, a square and a pentagon...

(If you find yourself a little rusty, you can also download a dictionary of mathematical words from the www.mathsgogogo.co.uk site). Also try to use the 'correct' words for concepts as soon as you feel your children can handle them ('subtraction' for 'take away' and 'multiplication' for 'times', to give just two examples).

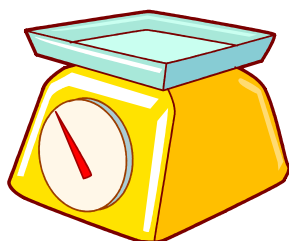


Find strategies for coping if you have children of different ages. When you try to carry out some of the activities in this document with one of your children, you will find the other will soon become interested and try to join in which could interfere with the learning of the first. If the second child is older he/she will want to give the answer before the younger has had a chance to work it out for himself/herself. If younger, he/she will possibly try to wreck any experiment you have set up because he/she does not understand the significance of what you are doing. One way to deal with this is to have a task already prepared for the second child that is suitable for his/her ability level and age.

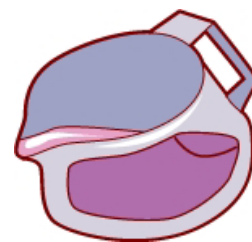
As with reading a good story, you should **stop before boredom sets in**, even if your child wants to carry on. This is a good way to get them to continue next time when they are fresh. You will have to make a judgement call when this should be, depending on the age, level and powers of concentration of your particular child. Learn from your experience.

If possible, **have an 'activities area'**. This is not always possible, especially if you live in a flat or small house, but is often surprising how you can get around this. A large box containing all the items you need that may be stored away after use can be a great help. Perhaps you have access to a greenhouse that may be used for some of the tasks? It's really a matter of using your imagination.

Work entirely in metric units.



grams



millilitres

Do not worry about the mess some of the activities described here may cause. Rather than having a go at a child for spilling water or flour or soil, it is much better to demonstrate how these things may be kept under reasonable control in future and how to clear up properly after an activity.

Children are normally much more willing to clear up if you help them. Another thing that may sometimes prove useful is that children are more willing to do a job if you ask them just before bedtime! (I'll leave that one to you to manage.)

Hey, Mum! Look what I'm doing!



Don't try to push too far ahead of what your child's teacher is doing in school.

This will mean that your child will be bored in class and this will bring its own problems. It is much better to reinforce what has already been covered earlier in the year or in previous years or to choose topics that are not likely to be covered in school at all, but which will encourage a love of mathematics for life. If you talk to your child about the topics they are covering at school, you may be able to extend that to outside the classroom. For example, if you find he/she is studying the medieval period, you could help them do research to find the percentages of people that died at different ages ('How many died before one year old?' for example) and draw graphs of this information. Compare this with today's statistics. This is unlikely to undermine what the teacher is covering. In fact, the opposite is true. Your child could take his/her research into school and show the teacher. This would extend the understanding of all children in the class (and possibly the teacher's too!).

Don't worry if you feel your mathematics is not up to the job. Most teachers of mathematics will tell you that they have learnt more about how mathematics works since they started teaching it than they ever did at college. This certainly happened to me. If you find yourself thinking 'Wow, that's interesting, I never knew that!' you are only following in the footsteps of thousands of parents and teachers before you. Use these activities to extend your own mathematical knowledge where appropriate.

Ask questions. In fact, make this the norm rather than the exception. Instead of saying, for instance, "Now we know how this plant has grown, let's repeat the experiment with another plant," you could ask, "We've seen how this plant grows. What could we do now?" Then you could extend the questioning to, "How are we going to compare the results? How will we illustrate our results? How will we know which one has grown the fastest?" See – it all becomes second nature after a little practice!

Work with your children's teachers, rather than against them. I was a teacher for over thirty years and I can tell you it is a very difficult and tiring job and there is nothing a teacher loves more than a supportive parent with whom they can work to help the child. But also, there is nothing a teacher hates more than an aggressive parent who undermines (either knowingly or unknowingly) the work they are doing in the classroom. Bear in mind that most people have trouble coping with one, two or three children – teachers have to cope with about thirty and they deserve every bit of help they can get.



Although some of the principles demonstrated here may be extended to children above the age of eleven, I am really concerned in this document with children between five and eleven years old. I am going to assume that they have had plenty of the playgroup type activities such as blowing bubbles, playing with water and sand and pouring these from one container to another to see which holds more, lifting water and sand with crane mechanisms, handling paint and using crayons and pencils. (If your children have not had much experience of these activities, I suggest you start as soon as possible!) Let these activities continue as long as they are interested – there is so much to learn here about the nature of these materials and the containers that hold them.

In this document, I am going to describe a number of activities that you may wish to use with your children. Please don't think these are the only worthwhile ones. There are hundreds of things you can do and once you have the idea, you should look around for other activities that would suit your children. Try to get out of the house as much as possible. You could prepare an activity together to do in your local park and do this when the weather is fine (or when it's pouring down if you want to add an extra dimension to your work!).

For some of the activities, I am going to make a number of suggestions at different levels. As all children are different in age and ability, not all the suggestions would be suitable for your children. Older and more able children will quickly become bored with simple tasks, so with these children concentrate on the more difficult aspects of the topic (comparing results taken in different circumstances is one way of doing this). For younger and less able children simple activities with easy to comprehend outcomes are better. With a little experience you will soon be able to tell what will suit your children best. This will give you a little bonus that most other parents will not have. You will be in a much better position during parents' evening to discuss the progress of your children, isolate areas of difficulty and between you design strategies to help your children progress further.

Don't shy away from any areas of difficulty you have found. Remember that the first step in solving a problem is recognising that there is one. Try to isolate exactly the nature of the problems your children may have. It is then much easier to find a specific solution. For instance, it may be that you see that one of your children has great difficulty in performing long multiplication which other children of the same age and ability don't seem to have. Watch this child very carefully as this type of sum is worked and try to see what exactly the problem is. It may be that you notice he/she has no problem laying out the sum, knows where to draw the line between working and answer, knows how to carry figures into the next column and knows that a zero is necessary at the end of the second line of working, two zeroes at the end of the third line and so on. Despite this, a long time is still taken to calculate a sum. Then you realize the problem – your child does not have a good knowledge of tables! This really slows the pace in a complicated sum. If a child does not know that nine eights are seventy two, it could take a longer time to work that out than it does another child to complete a whole long multiplication sum. At least now you know the problem and if you cannot see how to resolve this issue, you can seek advice from the class teacher. By the way, there is a section later specifically devoted to tables.

Practical Experiments

Let's start with an example that illustrates how you may approach a task at different levels.

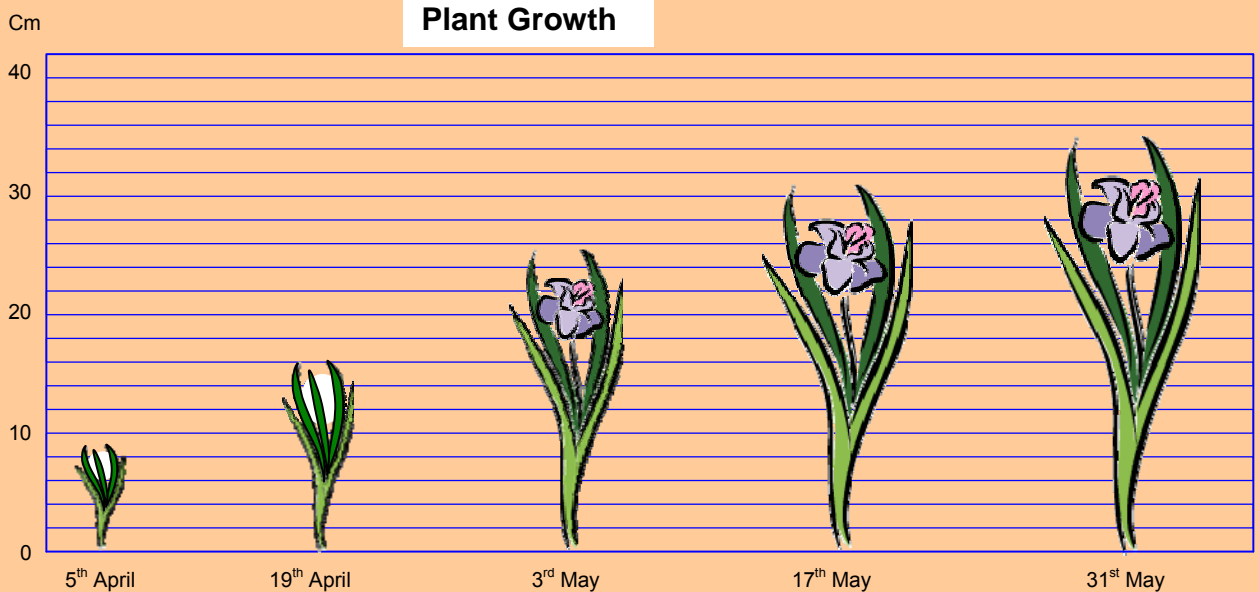
Grow plants and measure how they grow.

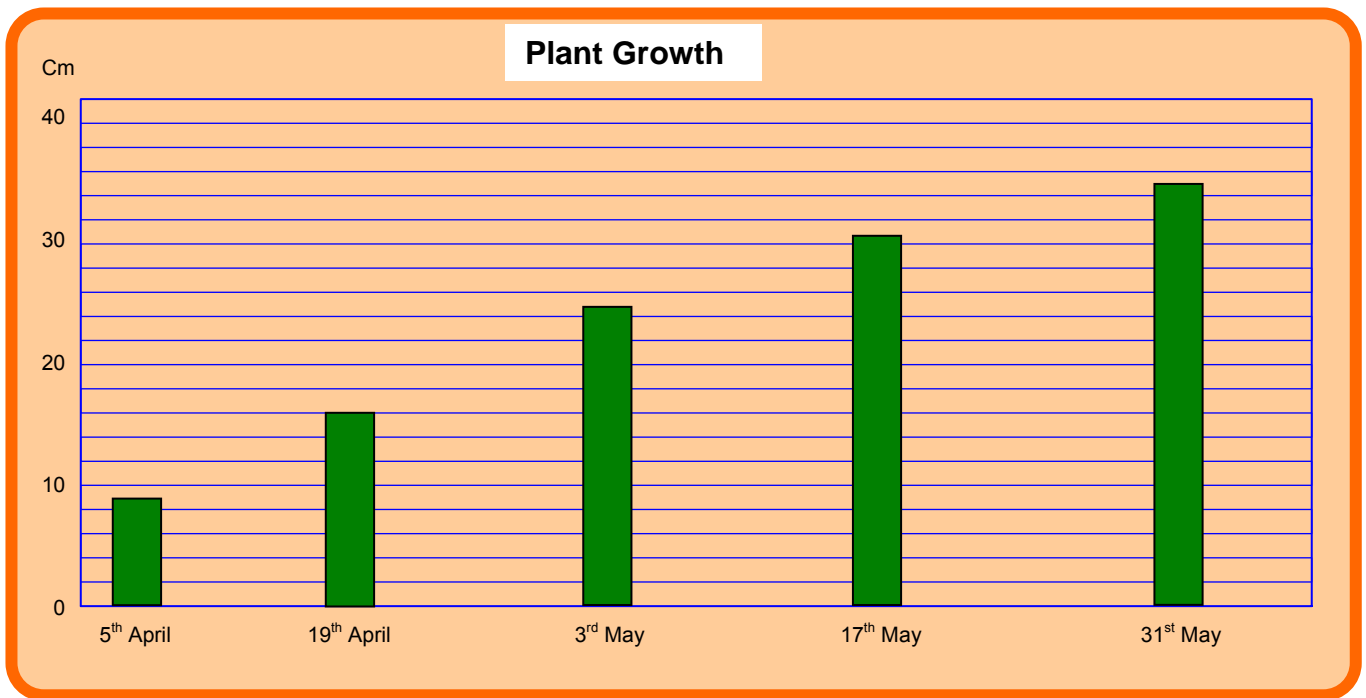
Simple level for younger and/or less able children: record one variable.

Grow a plant such as a tomato plant or runner bean either from seed or from a 'starter' bought in a garden centre. If a faster growing plant is required, mustard seed is very good.

Keep a record of how the plant grows. The easiest measurement of all to take is the height from the soil surface to the top of the plant. This can be recorded every few days and the data put into a table or graphical form. Some ways in which this can be done are illustrated below:

Date	Height (cm)
5 th April	9
19 th April	16
3 rd May	25
17 th May	31
31 st May	35





Children are often very surprised at how fast plants grow (so am I – especially the weeds!). This type of activity draws their attention to this fact as well as the mathematics involved.

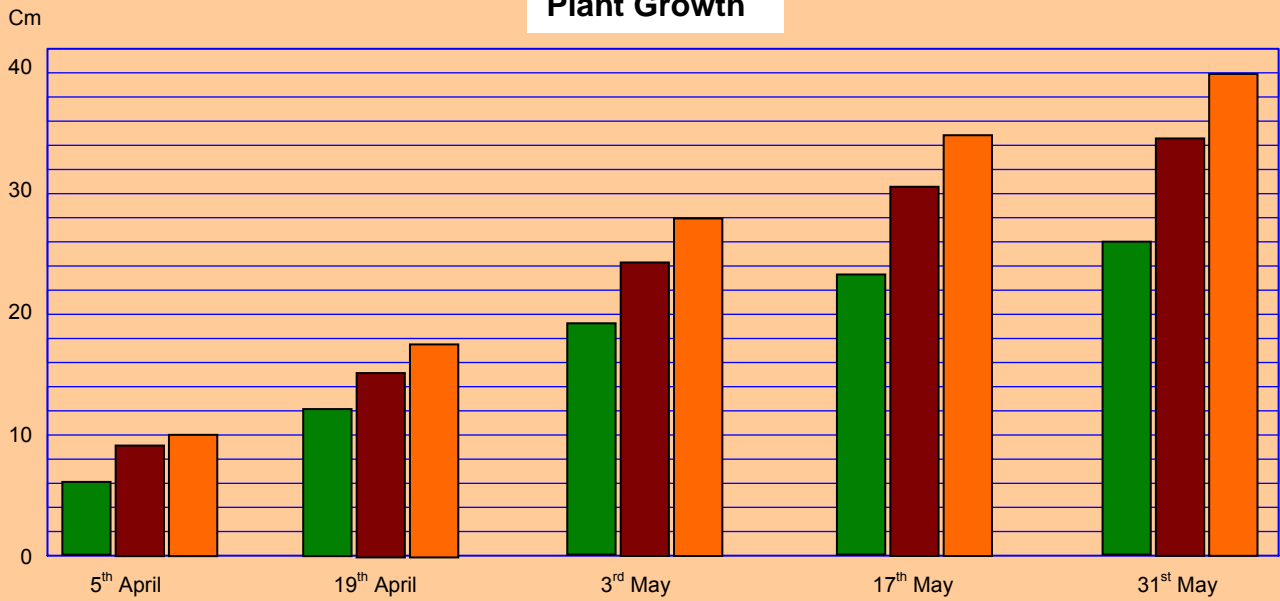
Intermediate level:
Making a comparison.

Grow several plants so that growth may be compared. This can be done by growing the same type of plant under different conditions. For example, they could grow one in a greenhouse, one in full sun outside and one in shade outside. Or they could give each a different, carefully measured amount of water each day, everything else being the same. Or they could grow three different types of tomato plant and compare the differences. Or the plants could be given different amounts of fertiliser, everything else being the same.

If growing tomato plants, wait until the plants are mature and then measure the amount of fruit each produces. Weigh each tomato or group of tomatoes in grams and keep a running total. This is a good exercise in recording masses as well as measuring height and length.

Find a good way of showing the results so that differences can be easily compared, such as:

Plant Growth



- Little water
- Medium water
- Much water

(If you have a way of measuring the water accurately, you can record here the exact amount – 50 ml, 100ml etc)

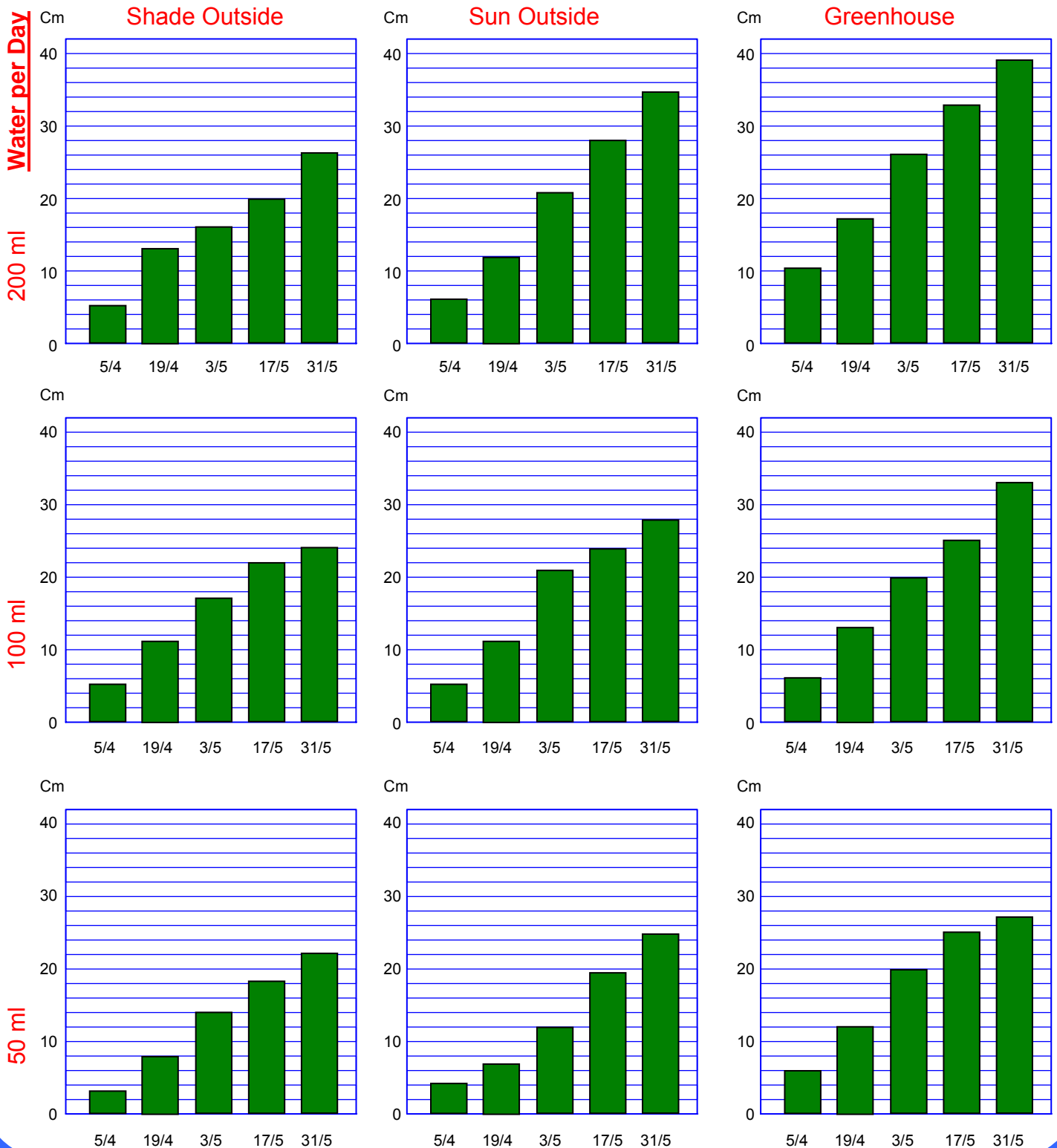
Mass of tomatoes produced

	Little water	Medium water	Much water
First crop	55g	76g	130g
Second crop	60g	95g	160g
Third crop	87g	120g	98g
Fourth crop	65g	87g	97g
Fifth crop		60g	
Total	267g	438g	485g

Higher level for older children or bright sparks:
Test two factors.

Change two variables at once. For instance, grow nine plants of the same variety of tomato and change the position and the amount of water. This may be illustrated as below:

Plant Growth



Results need to be recorded very carefully and conclusions drawn with even more care. Other things may be measured in this case such as the average leaf area of the mature plant (Choose three approximately average sized leaves for each plant and draw around them on squared paper. Find the average area and compare the nine plants.) Perhaps we would expect the plant with the most sunshine and the most water to have grown the largest, but is this in fact the case?

The final mass of the plant, once the soil has been shaken off, could also be measured and compared.

There are many variables that can be changed and measured, but children need to be careful about changing too many things at once. For example, if the plants grow so quickly that they have to be repotted, make sure they are repotted at the same time into the same size pots with the same compost. This is good practice for future science as well as mathematics lessons.

Having done all this, you may well want your children to write down the conclusions and they might not want to. I wouldn't push this too hard as they do a lot of this at school and you want this to be fun. Pushing them too severely will sooner or later put them off the whole idea of working at home! If this is a problem, content yourself with thinking how much mathematics they have learnt in doing these experiments and be happy with that.

Working in the kitchen

If you think about it, your kitchen is a mini mathematics workshop. You have facilities for

- weighing
- measuring the volume of liquids
- changing the temperature of liquids (both up and down)
- cooking (which involves a change of state)
- freezing (which involves a different type of change of state)
- cutting materials such as cheese and sausages
- grating
- producing bubbles

and a never ending supply of water and packets, cans etc of different shapes.

If you want to make full use of your kitchen as a mathematics workshop, you can also include a thermometer that reads from below 0 °C to over 100 °C (110 °C or 120 °C is ideal), a shape cutter and a measuring tape.

Make sure all the measuring instruments (scales, tapes, thermometers etc) measure in metric units.

I should say that all activities in the kitchen should be supervised by a responsible adult. Please don't leave young children alone with boiling liquids, hot ovens, cookers or sharp knives etc. Only ask them to perform tasks they are capable of performing safely. Children at school are used to being told that some jobs are dangerous and need to be carried out by an adult, so following the same procedure at home should not cause any problems. In any case, you are in charge in the kitchen and children should understand this.

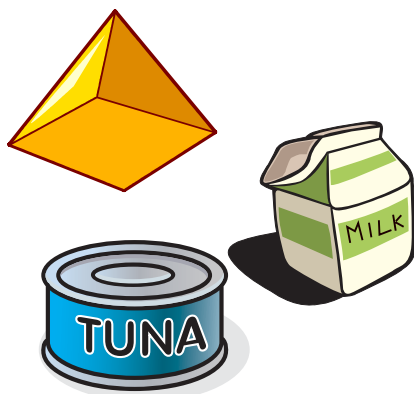
Here are some suggestions of activities that can be undertaken in the kitchen. There are many others, of course, I can only give a 'flavour' here (excuse the pun).



Sorting: Many things come into a kitchen in the course of a week and these are a great source of mathematical activity. Younger children love sorting. You could, for instance, pretend to accidentally tip all the fruit into your main bag and then you can ask your children to sort them for you. You can then ask, 'How many pears are there?', 'Are there more apples than oranges?', 'How many more apples than oranges are there?' You can, of course, do this with other things around the house – cutlery, socks, shirts etc.



Laying the table is a great activity because it teaches what we call 'one-to-one correspondence', i.e. the idea that each person needs one plate, one knife, one fork etc. You can also discuss this when getting dressed – each foot needs one sock and one shoe.



Older children can **sort foods** into fruit, vegetable, pasta and meat categories or the containers into cubes, cuboids, cylinders etc. (If you are not familiar with all the names of shapes, please download the mathematical dictionary from the www.mathsgogogo.co.uk site.) When you are trying to get a meal prepared in a hurry the children can be kept occupied by giving them some cans and packets from the cupboard to play with. They love to discover how to make towers and trains and cars. Okay, so the can may fall on the floor now and again – don't get angry, it's not the end of the world. When you can't bear it any more, give them some paper and ask them to draw the packets or draw what they have been making. Don't let them watch the television until you really, definitely and absolutely can't think of anything else for them to do and they can't either! Keep them away from that brain stifling box as long as possible. When you have a spare moment, have a look at this:

<http://www.poemhunter.com/poem/television/>.

Still older children can look at the labels and **put foods in order** of 'weight', 'price per Kg', 'amount of protein/sugar/fat etc per 100g' etc. Total calorie intake for a day's food can be calculated.

While you are doing this you could, of course, weigh members of the family and friends and compare yourself with the 'ideal' weight for your age and sex. You can find out what these are from sites such as

http://www.weightlossresources.co.uk/body_weight/healthy_weight/chart.htm, but don't forget to choose the metric options!!!!!!



Weighing: Get your child to hold two carrier bags and shut their eyes. In one put something quite heavy such as a pack of pasta. In the other put something light such as a tomato. Ask the child to estimate which is heavier and which is lighter. Do this with things that are closer together in mass. If the masses are very similar, you can check on the scales. Reverse the process so that the children put objects in your carrier bags and you have to guess which is heavier. They love to reverse roles to try to catch you out, but you can always word your questions so that they do most of the work: 'Was I right – is the cabbage really heavier than the lettuce?'

So many opportunities here. Weigh a potato. Weigh two potatoes and see which is the heavier (and by how much). Cut the larger potato so it weighs the same as the lighter one (just before cooking them for dinner, of course). Weigh some potatoes and then use your experience to judge the weight of other potatoes. Weigh these and see how accurate you were. Keep a table of your results. Does your accuracy improve with experience?

Does weighing potatoes help you judge the mass of tomatoes, packets of frozen peas etc?

Time: You can teach your little darling to tell the time in the kitchen. All you need is a clock on the wall. If they are still at the stage of learning the hours, keep your eye on the clock and as it approaches 4 o'clock or 5 o'clock etc, you can quite spontaneously ask them the time. Once they have mastered this, you can do the same with the half hours and then the quarter hours and so on.

You can also have a kitchen timer and teach them how to use it. Let them set it to boil an egg or time the pasta. If the pasta is too soft or the egg too hard make them eat it, they will soon learn! I am a great believer in learning from experience.

Don't forget that time also includes things such as dates, years, diaries and calendars which all represent sequences of events and children can fill in these events on calendars and write a diary.

Cooking: There are countless recipes that can be used and as you probably know far more about cooking than I, you are more qualified to choose them. Keep them appropriate to the age of the child.

The general idea is to get the children to do as much of the measuring and mixing as possible, so allow plenty of time for this and for the cleaning up afterwards. For younger children choose simple recipes such as gingerbread men, scones, mince pies and pancakes (you might like to do the tossing outside!). Rehearse in your mind before you begin the mathematics you are aiming to focus on. Cooking gives boundless opportunities for weighing ingredients and measuring liquids which can only be done to a limited extent in school. Getting your children to cook just one recipe a week will significantly increase the amount of practical measuring they do during their school life.

(Worried about the mess? Don't be. Make developing good working habits and cleaning up afterwards part of the process. Very young children don't even see clearing up as a job – to them it is just part of the activity as long as you are there to help and supervise. A few weeks of close supervision and they will accept it as part of the process.)

Younger children can use shape cutters to cut pastry to put onto pies, tarts etc. Teach them the names of the shapes as they do this. Then you can talk about the 'triangular pie', the 'circular pie' and so on.

Teach children how to read the scales correctly and particularly how to make sure they are reading zero before you begin weighing. Liquids are, of course, normally measured by volume, but there is no reason why they should not be measured by mass. One millilitre of water has a mass of one gram so 100 ml of water should have a mass of 100g (try it with your children and see). Don't forget to zero electronic scales with the empty container on before pouring in the liquid. With the older analogue scales (the type with a rotating needle) you will need to find the mass of the container and subtract this from the total mass to obtain the mass of the liquid (all good practice!).

With older children the recipes can of course be more sophisticated and other mathematical activities may be included. For instance, weigh the mixture for bread immediately before it goes into the oven and weigh it when it comes out. Calculate how much water has been lost in grams and as a percentage of the original weight.

Eg.	Mass before cooking	1955 g
	Mass after cooking	1675 g
	Mass loss (water)	280 g

$$\text{Percentage mass loss} = \frac{\text{mass loss in grams}}{\text{original mass in grams}} \times 100 = \frac{280}{1955} \times 100 = \underline{\underline{14.3\%}}$$

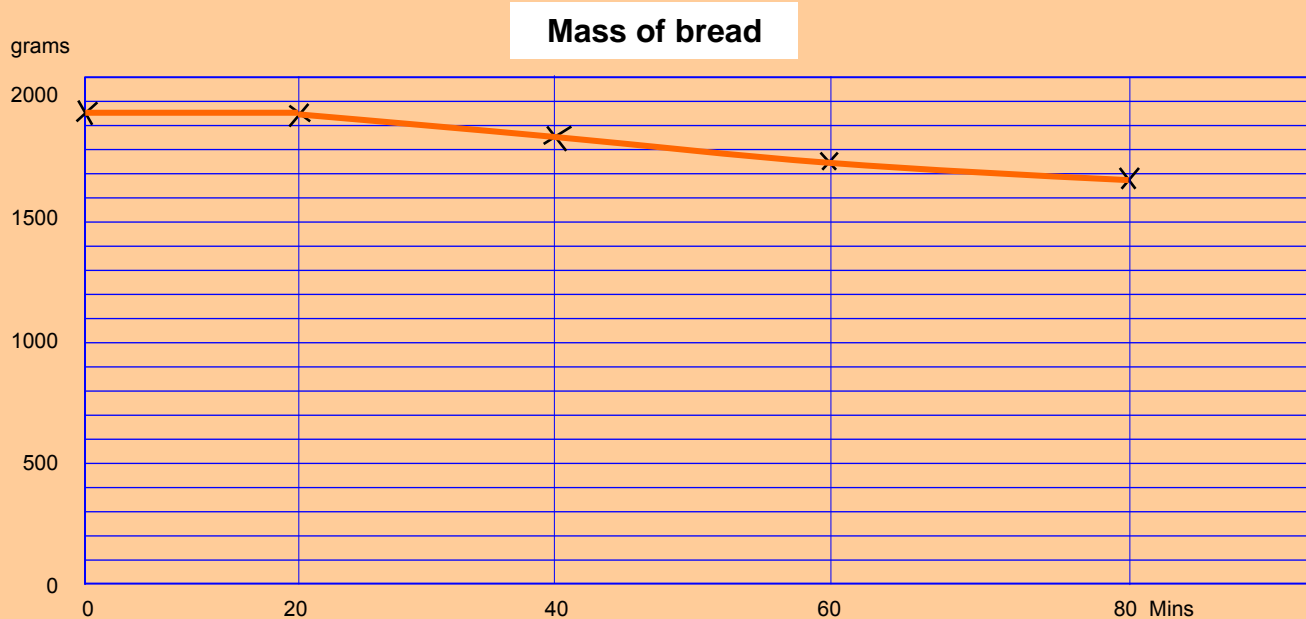
(This type of calculation does need a calculator.)

In other words, the bread is 14.3% lighter than it was before it went in the oven.

If it won't spoil the cooking process, it is possible to weigh the bread every, say, ten minutes, but please help the children with handling such a hot object. Draw a graph of the mass against time.

Recording Results

Younger children can draw a simple bar chart, but older children can draw a line graph and connect up the intermediate points. This should only be done when the intermediate points mean something. In this case they do, because the bread continues to lose water between weighings.



Line Graph showing intermediate values.

More Able Children

More able children can try all sorts of variations such as weighing out each ingredient and keeping a record. See if the total mass of the mixture before going into the oven is exactly the same as the total of the masses of the individual ingredients. Account for any differences. (These are normally due to errors in the accuracy of the scales or errors in reading the scales.)

Keep a special note of the mass of water going into the bread. See how much of the water has evaporated. In the above example, suppose 450 g of water (450 ml) was put into the bread and 280 g was lost through evaporation, the remaining 170g must still be in the bread. What has happened to it?

Other Kitchen Activities.

Measure the temperature in different parts of the refrigerator. To do this properly, you need to put the thermometer in each place, close the door and wait a few minutes for the temperature to adjust. Open the door and read the temperature as quickly as possible. Is it true that cold air sinks and warm air rises?

Repeat this exercise for the freezer if your thermometer goes down far enough.

Put some water in a container. An old ice cream container is perfect for this. Put the thermometer in the water and read the temperature. Put the whole lot in the freezer and read the temperature at intervals of, say, twenty minutes. Note the temperature at which the water freezes and note how long it stays at that temperature. You need to be careful here because the pressure of the freezing ice could break a weak thermometer and it is probably better to remove it at the last moment. You could then wait until it is completely frozen, make a small hole in the ice and insert the thermometer into this hole, carrying on with the temperature measurements for some further time. You should find that the water drops in temperature quite quickly, but holds at about 0°C for a long time while the water freezes. It should then continue to cool down until it reaches the temperature of your freezer.

Record the readings and draw a graph as described above for the mass loss in the bread.

You can repeat this experiment in reverse. Freeze a block of ice overnight. Take it out in the morning, make a small hole in it and insert the thermometer. Record the temperature over the next few hours every twenty minutes and draw a graph as before.

You can also measure the volume of the ice compared to the volume of the water once it has melted. If you use an ice cream or similar container, the water is restricted in the horizontal directions as it freezes and so the change in volume will be represented by a change in height. Is it higher when the water is frozen or when it is liquid?

Which has the greater mass, the water in liquid form or the same water when frozen?

These experiments involve very hot water so definitely need to be supervised by an adult.

Fill a pan about two thirds full of water straight from the tap and heat it on the hob. Put a thermometer in and read the temperature every, say, two minutes, keeping a record on paper.

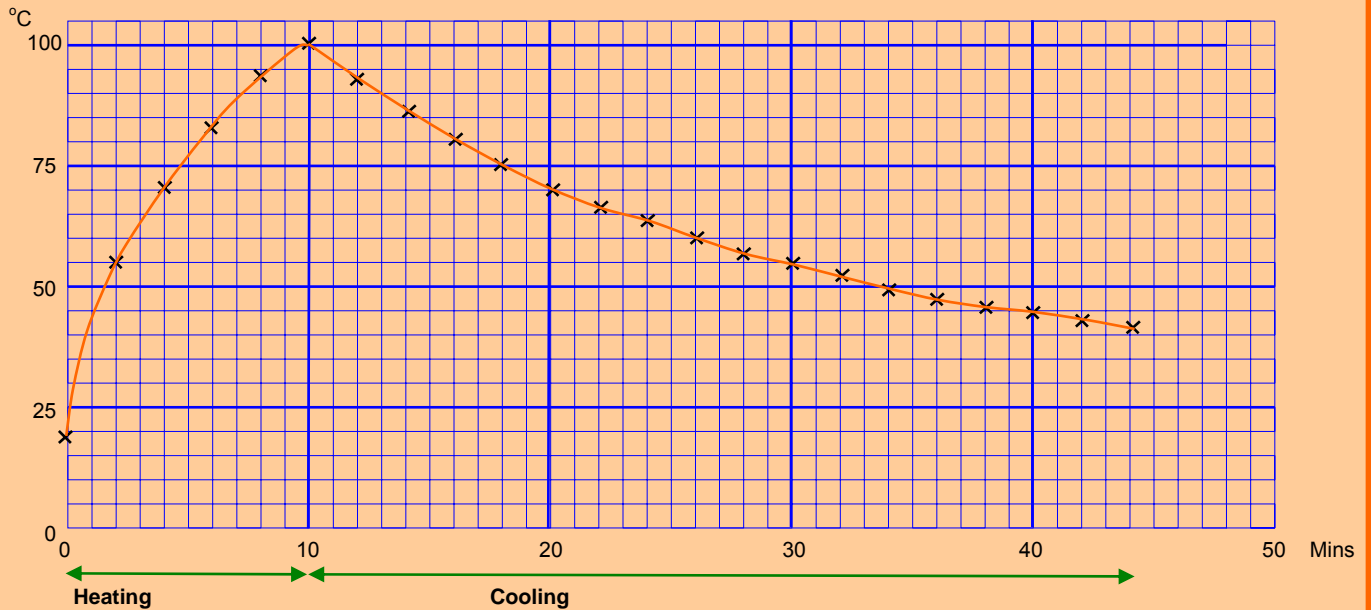
Plot the temperatures on a graph. The graph should start looking as though it is going to be a straight line, but as you plot more points, it will begin to curve over, because the hotter the pan becomes, the faster heat is lost, so it will take longer for the temperature to rise from 60°C to 100°C than it will from 20°C to 60°C , even though both temperature differences are 40°C .

You can now let the water cool and continue to measure the temperature. It normally takes longer to cool down, and this is reflected in the curve drawn (see below).



You could then put both sets of results on the same graph.

Temperature of Water in a Pan



This experiment can be repeated using half as much water. The results can then be compared. Did half the amount of water heat up and cool down twice as quickly? How can you tell?

When the temperature reaches 50°C carefully put some ice cubes into the water and observe the effect on the temperature. Repeat this during the cooling phase. Can you predict how it will affect the graph before you carry out this part of the experiment?

Bubbles. Who can produce the largest bubble in a sink full of water and washing up liquid? How are you going to measure the bubbles to see who has the largest?

If you have a bubble blowing device, can you blow the largest bubble? How will you measure it?

What happens to the shape of bubbles when you put several together? Can you think of an animal that uses this effect? (Answer: bees when making their honeycomb.)



Number Activities

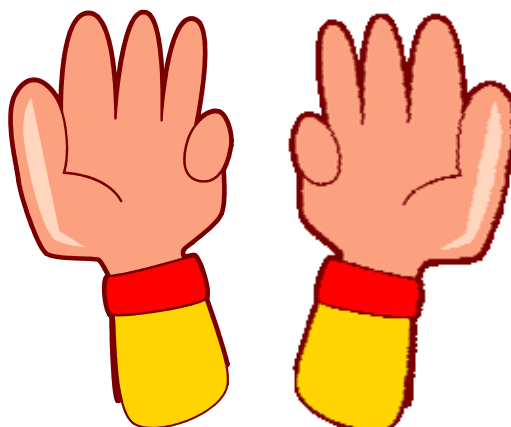
Some people would argue that work with numbers is one of the most important areas of mathematics, especially in the primary years, and there is no doubt that any child who is proficient with all aspects of number work will be well placed to work in other areas of mathematics such as working with data, drawing graphs and calculating angles.

It is surprising how little extra work is needed to accelerate a child in this field if the correct approach is taken. This is to make it fun and appropriate for the individual child. Here I give a range of activities that will help children develop the appropriate skills, but don't forget, these are only ideas and once you begin to see number all around you, hundreds of ideas will spring into your mind.

The activities are very wide ranging in scope and difficulty because there is a very great range of ability between the children at age five who need some extra help and those at age eleven who are very bright and need stretching, so choose the ones that are right for your children and make the progression match the improving ability of your children. At all times remember to keep it fun.

Give Me Five

I am sure you are familiar with the game in which one person says to another, "Give me five!" and they slap the palms of their hands together. It is very easy to adapt this to "Give me four!" or "Give me two and a half!" I have two nephews, one four years old and the other six. They have great fun with this one. You can, of course, extend this to two hands: "Give me eight ... six and a half ... five and a quarter!" or if you are playing by the paddling pool, "Give me twelve ... fifteen and a half ... twenty!" If you incorporate this sort of nonsense into your everyday routine, children see it as a game and like to rise to the challenge just as they like to score a goal in a game of football.



Car number plate games

My children used to love these games and now they are all geniuses like their dad, so there must be something in them!

You are driving around in your car. There are hundreds of other cars on the road and every one has a number plate. Unfortunately, the newer registrations do not have three digits, but there are still plenty of older cars around that do.

Variation 1. Add the digits as quickly as possible.

R378 FRE



$$3 + 7 + 8 = \underline{18}$$

Variation 2. Spot a three digit number plate. Read the first two digits as a number and add the third digit.

R378 FRE



$$37 + 8 = \underline{45}$$

Variation 3. Spot a three digit number plate. Read the first two digits as a number and subtract the third digit.

R378 FRE



$$37 - 8 = \underline{29}$$

Variation 4. Add a fixed number to a number plate number. Use ones appropriate to the children's age and ability such as "add 1/2/3" and then progress to "add 10/20/30" or "add 100/200/300" and then to "add 25/35/105" etc as appropriate.

R378 FRE



$$378 + 30 = \underline{408}$$

Variation 5. Double a number plate.

T175 DDS



$$175 \times 2 = \underline{350}$$

Variation 6. Halve a number plate.

V455 UHT



$$455 \div 2 = 227.5$$

or $\underline{227\frac{1}{2}}$

Variation 7. Add two number plates together.

R378 FRE

M277 MAS



$$378 + 277 = \underline{655}$$

Variation 8. Find a relationship between the three digits in the number plate. Sometimes it is impossible to find such a relationship. If this is obviously the case, quickly choose another!

M277 MAS



There are 2 sevens in
the number

P725 DON



$$7 - 2 = 5$$

Or $7 = 2 + 5$ etc

J238 TYG



Three 'twos' multiplied
together come to eight
($2 \times 2 \times 2 = 8$)
or (more mathematically)
 $2^3 = 8$

You will probably soon find several other ideas.

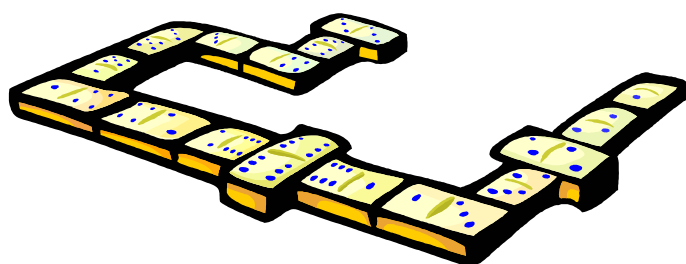
Play similar games with any source such as **house numbers**. Young children can learn much about numbers by simply walking down the street – odd numbers are usually on the left of a road and even numbers on the right, but there are exceptions as I discovered to my cost some time ago when I walked a very long way down a road before I realised that the numbers went from 1 to about 500 down one side and 501 to about 1000 back down the other!

I once lived on a large estate where there are no houses numbered 13 (this is quite common and the reason can be discussed) and the road I now live in starts at number 3, has no number 13 or 15 on one side and nothing between 14 and 24 on the other. The reason for this is a mystery I have yet to solve.

Once the correct pattern has been spotted, young children may be asked to predict the number of the next house in the road. Once they can do this for small numbers, take them to a much longer road where the numbers go way past 100.



When children today talk of playing games, they normally mean playing games on a computer, but the old fashioned games such as snakes and ladders, cards, games of dice, dominoes and so on provide much practice in mathematics and all children should be playing them often. They also provide opportunities to socialise within the family and friendship group and teach children to be a good loser as well as a good winner.



Most people know how to play these games and there are many descriptions of card games such as pontoon and whist on the internet if you don't. I won't describe them all here, but I wanted to say how important I believe these games to be.

Dominoes also provide opportunities for other mathematical activities and I shall return to these later.

Blank playing cards. These are playing cards with nothing on them. They come in two varieties – completely blank and blank on one side and a normal design on the reverse side. Normally, the completely blank cards are good enough. You can, of course, cut these out of plain white card yourself, but I really must encourage you to purchase some professionally produced cards because they work out cheaper in the long run, save you a lot of time with the marking out and cutting and are manufactured to the same standards as normal playing cards. If you type 'blank playing cards' into Google, you will find many companies that sell these. If you live in the UK you may like to try -

<http://www.spiritgames.co.uk/playdis.php?selection=Pl1ying%20Cards>

I should point out I have no shares in this or any other card manufacturing company. Once you start using these cards, you will soon wish you had bought more than one pack, so why not buy several packs or a large box to begin with?

You will also need a few felt pens to add numbers, shapes etc as desired.

Here are some suggestions for games to play with these cards..

Take a set of 28 cards and look at the display below. You need to make all the A's have the same result, all the B's the same result and so on.

A	A	B	B	C	D	D	G
A	B	B	C	C	E	E	E
A	C	B	D	C	F	E	F
A	D	B	E	C	G	E	G
A	E	B	F	D	D	F	F
A	F	B	G	D	E	F	G
A	G	C	C	D	F	G	G

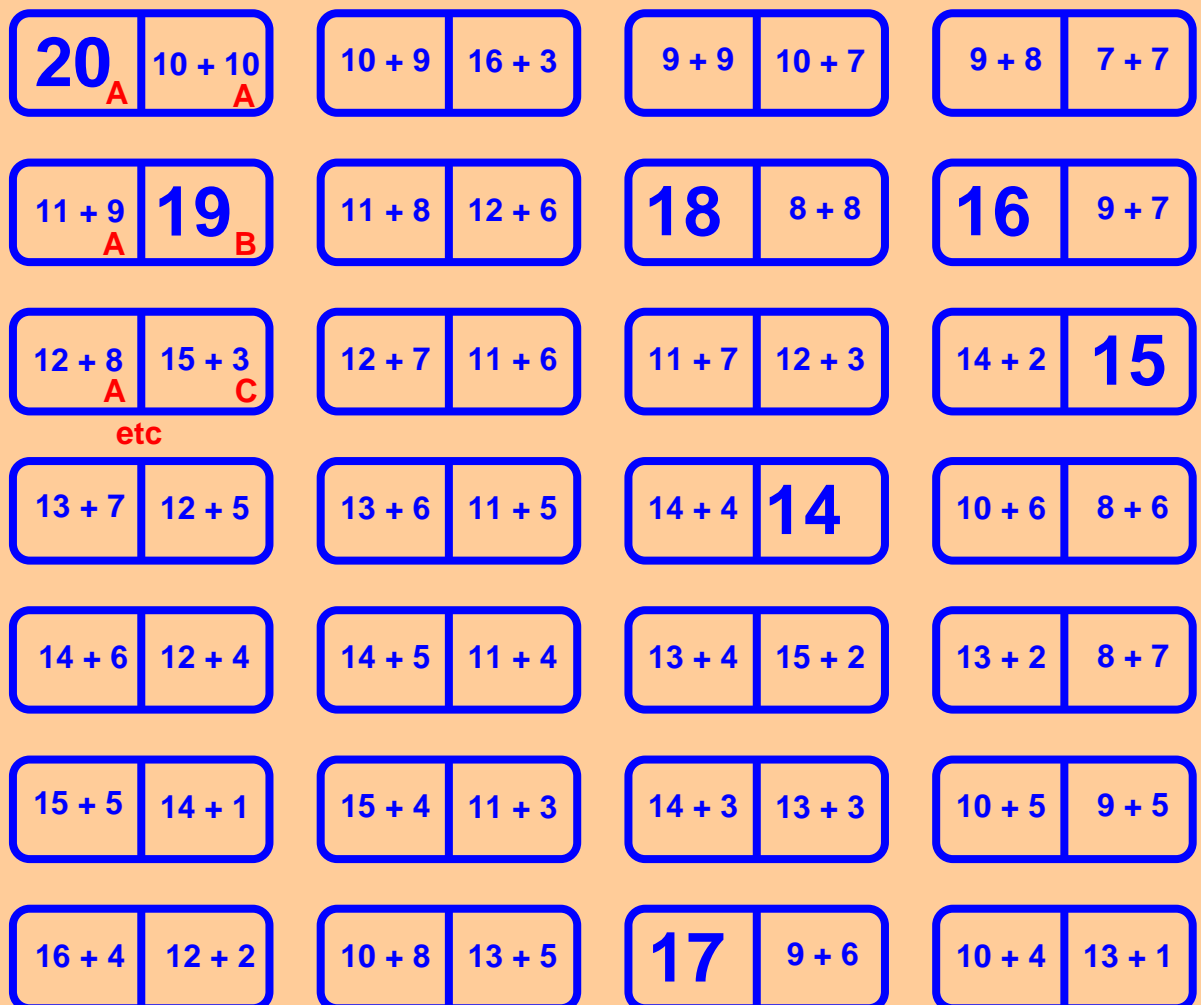
As an example to see how this works, let's assume you want your children to practise addition with **totals up to 20**. You need seven totals, one each for A, B, C, D, E, F and G, so let's have:

$$A = 20 \quad B = 19 \quad C = 18 \quad D = 17 \quad E = 16 \quad F = 15 \text{ and } G = 14$$

The next thing to do is devise several ways of writing sums that give these answers. Ideally, you want eight for each number, but sometimes this is virtually impossible, so you can use the same ones two or three times if necessary. (Here we have plenty of choice, so duplication won't be necessary in this case.)

We choose for A: 20, 10 + 10, 11 + 9, 12 + 8, 13 + 7, 14 + 6, 15 + 5, 16 + 4
 for B: 19, 10 + 9, 11 + 8, 12 + 7, 13 + 6, 14 + 5, 15 + 4, 16 + 3
 for C: 18, 9 + 9, 10 + 8, 11 + 7, 12 + 6, 13 + 5, 14 + 4, 15 + 3
 for D: 17, 9 + 8, 10 + 7, 11 + 6, 12 + 5, 13 + 4, 14 + 3, 15 + 2
 for E: 16, 8 + 8, 9 + 7, 10 + 6, 11 + 5, 12 + 4, 13 + 3, 14 + 2
 for F: 15, 8 + 7, 9 + 6, 10 + 5, 11 + 4, 12 + 3, 13 + 2, 14 + 1
 for G: 14, 7 + 7, 8 + 6, 9 + 5, 10 + 4, 11 + 3, 12 + 2, 13 + 1

And then we put them on the cards as in this diagram, mixing them up as we go, otherwise we tend to get some dominoes with two difficult sums and some dominoes with two very easy sums.



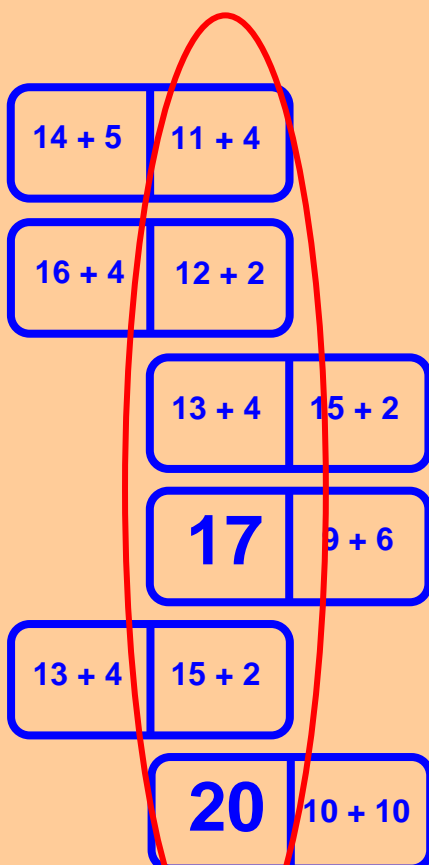
Now you can play dominoes in the traditional way.



Notice this one needs to be placed upside down.

Apart from the traditional game of dominoes, you can ask questions such as, “Is it possible to arrange the dominoes in one long line in such a way that **all** the dominoes are used and, if it is possible, is there any relationship between the numbers at the very beginning and at the very end of the line?”

All this will obviously be done in their heads. If you want them to have a little calculator practice, you can ask them to arrange the cards in a vertical line so that one column totals as near to 100 as possible:



Totals 100

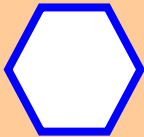
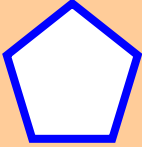
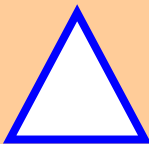
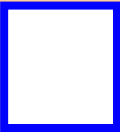
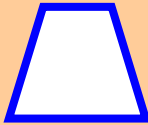
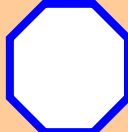
You can make up sets of cards with different questions such as multiplication and subtraction. With multiplication, you may find you have more questions for certain answers.

This is simply because some numbers have a lot more factors than others.

The number 34 can only be made from 1×34 and 2×17 , whereas 36 can be made from 1×36 , 2×18 , 3×12 , 4×9 and 6×6 .

This is not a problem – just repeat the questions for those numbers with fewer factors, perhaps writing them in numbers and in words. The important thing is to get your children practising. If you are having problems making a complete set, remember that you don't have to use fifty two cards – any convenient number will do.

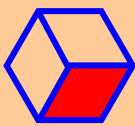

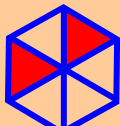
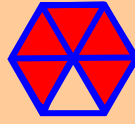
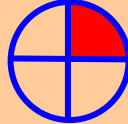
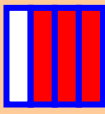
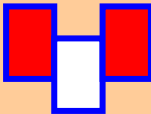
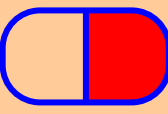
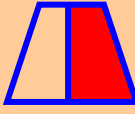
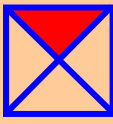
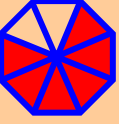
You can also match **shapes to their names and number of sides** and mix them up like this:

Hexagon	Five Sides	Three equal sides	
	Only one pair of parallel sides	Octagon	Four equal sides and four right angles
			

If you are not sure about the properties of shapes, you can download our dictionary from www.mathsgogogo.co.uk and look at the section on 'Two Dimensional Shape'.

Now I am sure you can see what I mean about buying quite a few of these blank cards – you could think of a new idea almost every day!

You can also match **fractions**:

One Third				One quarter	$\frac{1}{3}$
One Half	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$		
$\frac{5}{6}$					Five Sixths
$\frac{1}{4}$			Two thirds	Three Quarters	One half

Fractions, decimals and percentages.

As every percentage has a corresponding decimal value and a corresponding fraction, it is easy to make cards to practise these equivalents:

0.4	25%	One Quarter	0.5	Two Fifths	$\frac{2}{5}$
One Half	$\frac{1}{2}$	40%	$\frac{3}{4}$	Three Quarters	20%
0.2	75%	0.25	One Fifth	$\frac{3}{5}$	$\frac{1}{5}$
$\frac{1}{4}$	0.75	60%	Three Fifths	0.6	50%

As Spock used to say, "There are always possibilities"!

TABLES

Probably the most commonly asked question of a candidate for a mathematics teaching post in a primary school is, 'Do you believe children should know their tables?'

Of course children should know tables. How on earth can they hope to calculate anything but the very simplest of problems if they do not?

'Calculator,' did I hear you say? Well, calculators have their place, of course, and I shall be covering this topic later, but for now let me ask, 'What are children going to do if there is no calculator available?' We need to remember too that it is often the case that a child who knows their tables and other numerical skills will out-perform a child using a calculator in most calculations because it actually takes quite a long time to press all those keys. Generally speaking, calculators should be kept for calculations that cannot be done in one's head or on paper.

So, tables. To learn them parrot fashion or not to learn them parrot fashion, is that the question? Actually, no, because by the time you come to help them at home, you are either trying only to speed up table recall or you have just realised your child has a problem. Your child's teachers should have already done a lot of the groundwork.

The most important thing to do is to isolate the table facts that your child is finding difficult and I am going to show you here how to do that. I am going to assume your child is over about eight years old, by which time they should have a good grounding of (or 'know', as most people like to think of it) all tables up to 10×10 . If your child is younger, apply what I am going to say next, but adapt to his/her particular level.

Here is a full table square:

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

At first it seems as though there are 100 facts to know (5×6 , 7×8 etc), but actually there are far fewer than 100. First, draw a diagonal line from top left to bottom right. The table is symmetrical about this line, which gives us the fact that seven eights is the same as eight sevens etc. For the purpose of counting the number of facts to be learned, therefore, we can discount those in the yellow boxes.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Let's look at what is left. The one times table is pretty obvious and most children learn the ten times table pretty easily, so let's discount them too. And, to be honest, any child who does not know the two and three times tables should not really be attempting the 6, 7, 8 and 9 times tables, so we can pretty safely also eliminate the two and three times tables.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

So now we are left with just 21 what you might call 'Tricky Dickies' and, believe it or not your children may already know some of these.

In summary, we can say that any child who has a good knowledge of the simpler tables and knows the Commutative Law (i.e. that $3 \times 4 = 4 \times 3$ etc) will have at most 21 multiplication facts to brush up on and it is almost certainly these twenty one that are causing all their tables problems.

(The Commutative Law can be demonstrated quite easily with a simple diagram. Suppose we wish to show that six fives is the same as five sixes. Draw a rectangle of crosses six by five:

```

X X X X X X
X X X X X X
X X X X X X
X X X X X X
X X X X X X

```

This can be thought of as five rows of six crosses or six columns of five crosses. In both cases the number of crosses is thirty.)

Now, this is what you need to do. Give them a table square to fill in like the one below in which the numbers across the top and down the left side are arranged randomly. (By the way, if you become a member of the www.mathsgogogo.co.uk site, you will have access to many of these as well as thousands of excellent worksheets and hundreds of interactive activities to use with your children to improve their mathematics generally).

x	8	2	7	1	10	4	6	3	9	5
5										
4										
10										
2										
6										
7										
1										
8										
9										
3										

Ask your children to complete the table square. Don't worry about the time it takes (try to be very patient and encouraging), what we are trying to find out here is which table facts they don't know – speeding them up is a different issue I will deal with in a moment.

When they have finished, send your child off to do something completely unrelated while you sit down and have a really good look at how they have done.

Put a ring around the ones they have calculated incorrectly. You will almost certainly find your child falls into one of three groups:

a) There are so many mistakes, even of the easy ones, that they obviously have no real knowledge of tables at all. If this is the case (assuming your child is of the age when they would be expected to have this knowledge), there is not a great deal I can do here. Your child needs much one-to-one help and the best thing you can do is to make an appointment to see your child's mathematics teacher and together develop a programme that will help to fill in the gaps. As you have taken the trouble to read this far, I think I can assume that you would be prepared to work with your child at home (if only more parents were willing to do this!) so try to develop a programme in which you help your child for a few minutes every evening under the guidance of the teacher and build in times when you can meet again for a few minutes to discuss progress.

b) Your child shows a good understanding of the simpler tables (1, 2, 5 and 10) but makes some mistakes with these. Generally they make a complete hash of the difficult ones. Start by asking them orally to give you the answers to the ones with which they have made mistakes in the 1, 2, 5 and 10 times tables. If they can answer these easily and correctly, don't worry further about them as all children make occasional mistakes. Read the next section instead.

If they are not able to answer these simple ones correctly, forget the more difficult ones for the time being and practise these simpler ones until they know them really well. Don't forget to test them in reverse too. Say, "What is three times six?" As soon as they have answered say, "What is six times three?" This reinforces the Commutative Law. (Remember children do not need to know this term, just how it works).

When you think they have mastered them well enough, test them again with a full table square and put a ring around the errors as before. Then move on to the next section.

c) Okay, now we are down to the remaining 21 Tricky Dickies and you may find they know some of these already (surprise, surprise!).

First of all, discuss with them the Commutative Law and make sure they really understand this. Say things such as, "You know three fours is twelve *means* that four threes is also twelve, don't you. Well we can use the same idea with the more difficult ones – if seven eights is fifty-six, then eight sevens must be fifty-six too, yes?" (If they cannot see this, draw a diagram similar to the one on page 30.)

If you are lucky, there will be a glimmer of a smile as your child suddenly realises that this small fact is going to save them an awful lot of work.

Now it is easy. Make a list of the ones they do not know and go through them every evening for as long as it takes. Start with the hardest ones, using the principle that, “If you can get this one, the rest just get easier and easier.” There’s something psychological about jobs that get easier being more satisfying than jobs that get harder (in a child’s mind, at least).

Start with, say, four or five of the hardest and practise these until they really know them. Then just use these for revision each evening to keep them fresh in their mind and move onto the next four or five. After a surprisingly short time, you will have covered them all and you will have a child who ‘knows his/her tables.’ Give yourself a star!

The next thing is to speed them up. Give them blank table squares like the one above with the numbers across the top and down the side arranged randomly and time them to see how long they take. Do this, say, twice a week and keep a record of the times to the nearest second. To start off they will take fifteen minutes (half an hour, all night....), but will soon begin to speed up and that is where the pay-off comes. All that effort is now beginning to show real dividends – as your child becomes faster they become more confident and really look forward to doing the table square (yes, it really is true – with children success definitely breeds success). Not only that, they will also feel more confident in the classroom because it is surprising how many times tables are needed in mathematics lessons (you only need to look through this document to appreciate that) and they will see themselves creeping ahead of their classmates. Hopefully their teacher will also notice a difference and make suitably encouraging comments.

By the way, there is a limit to how fast it is possible to complete a table square simply because of the time it takes to write in the answers. A really excellent child of about eleven years old will be able to complete the whole table square in less than two minutes, but this is really exceptional; a time of five minutes is very good for most older children and please be happy if your child achieves this – it is much better than most children can do!. Look at the improvement your child has made and focus on congratulating them for that. (Just in case you want to try it yourself as a parent, my best time was about one minute twenty seconds – please let me know if you can beat that).



Equivalent fractions

Most people think of equivalent fractions as a sub-topic of fraction work just as adding or multiplying fractions are, but an understanding of equivalent fractions is much more fundamental than that since equivalent fractions are the key to just about everything involving any calculation or comparison work with fractions.

Let us first take a trip down Equivalent Fraction Lane to see why this is so.

What are equivalent fractions?

Equivalent fractions are fractions that look different, but actually have the same value. The easiest example of all is the two fractions $\frac{1}{2}$ and $\frac{2}{4}$. Two quarters of a cake is obviously the same amount as one half. Three sixths ($\frac{3}{6}$) is another fraction that is equivalent to one half.

We can build a whole group of equivalent fractions by using two processes – [lecnacing](#) and [cancelling](#).

Lecnacing: This is the process of starting with small numbers in the numerator (top number) and denominator (bottom number) and multiplying both by the same whole number.

$$\text{Eg. } \frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \qquad \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \qquad \frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$$

So $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ are equivalent fractions.

By continuing to lecnac by 5, 6, 7 etc (i.e. multiply both the numerator and the denominator) we can obtain as many equivalent fractions as we wish.

Starting with another basic fraction, say $\frac{2}{3}$.

we can lecnac by 2, 3, 4, 5, 6, etc to obtain:

$$\frac{2}{3} \rightarrow \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \dots$$

Cancelling: Cancelling is the reverse of lecnacing. Instead of multiplying the numerator and the denominator by the same number, we divide both by the same number. This process is used to simplify numbers with large numerators and denominators.

Eg. $\frac{45}{55} \div \frac{5}{5} = \frac{9}{11}$, so $\frac{45}{55}$ and $\frac{9}{11}$ are equivalent fractions.

Using Equivalent Fractions.

These processes would not be of much use if we did not need them in the operations involving fractions, so let us take an addition sum as an example.

Let us say we wish to calculate $\frac{5}{6} + \frac{3}{4}$

The first thing we need to do is make the denominators of both fractions the same. We notice that if we multiply the denominator of the first fraction by 4 we get 24 and if we multiply the denominator of the second fraction by 6 we also get 24. So we must lecnac the first fraction by 4 and the second by 6, so here goes:

$$\frac{5}{6} + \frac{3}{4} = \frac{20}{24} + \frac{18}{24} = \frac{38}{24} = 1 \frac{14}{24}$$

But $\frac{14}{24}$ can be cancelled by 2 to get $\frac{7}{12}$, so the final answer is $1 \frac{7}{12}$


Now, here's the rub: $\frac{5}{6}$ and $\frac{20}{24}$ are equivalent fractions.
 $\frac{3}{4}$ and $\frac{18}{24}$ are equivalent fractions.
 $\frac{14}{24}$ and $\frac{7}{12}$ are equivalent fractions.

So we have used the concept of equivalent fractions three times in this relatively simple sum. We also use them when we subtract, multiply and divide fractions and when we compare fractions.

If you feel a little shaky with this work (it may be some time since you did this in school yourself), you can leave the processes and technicalities to your children's teachers, but what you can do is make sure your children are incredibly good at Equivalent Fractions and here's how:

Take 48 blank playing cards and divide them into four 'suits'. Use the same colour ink for all 48 cards. On the first suit put a set of twelve equivalent fractions, on the second suit another set and so on, like this:

Basic Fractions Lecnac by 2 Lecnac by 3 Lecnac by 4 Lecnac by 5 Lecnac by 6 Lecnac by 7 Lecnac by 8 Lecnac by 9 Lecnac by 10 Lecnac by 11 Lecnac by 12

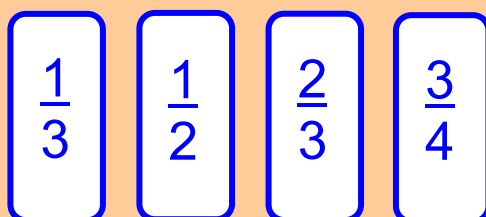


$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$	$\frac{11}{22}$	$\frac{12}{24}$
$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	$\frac{5}{15}$	$\frac{6}{18}$	$\frac{7}{21}$	$\frac{8}{24}$	$\frac{9}{27}$	$\frac{10}{30}$	$\frac{11}{33}$	$\frac{12}{36}$
$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$	$\frac{22}{33}$	$\frac{24}{36}$
$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$	$\frac{18}{24}$	$\frac{21}{28}$	$\frac{24}{32}$	$\frac{27}{36}$	$\frac{30}{40}$	$\frac{33}{44}$	$\frac{36}{48}$

Then try these activities:

1. **Sorting.** Ask your children to sort the cards in order of size. Because all the cards in a particular suit are equivalent (i.e. have the same value but look different), there should only be four piles, but don't tell them that before they have tried it.

With the fractions given above, the order should be:



with all the other cards in each suit underneath these four or, since they are equivalent, on top or some under and some above!

I would suggest you do this every time you use the cards and see if the children can perform this task quicker each time.

If you are not sure how to see which order to put them in, there are two ways. One is to use a calculator. Divide the numerator by the denominator for each fraction and write down the answers. Then compare the decimals. In this example:

$\frac{1}{3} = 0.3333333...$, $\frac{1}{2} = 0.5$, $\frac{2}{3} = 0.6666666...$, $\frac{3}{4} = 0.75$ and you will see that they are in the correct order.

By the way, you may have noticed in doing this that a fraction can be thought of as a division sum. In fact, it turns out that as the children get older, this is a better way of thinking of fractions.

The second method is to change the fractions to equivalent fractions with the same denominators.

Eg: $\frac{1}{3} = \frac{4}{12}$, $\frac{1}{2} = \frac{6}{12}$, $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$

2. **Snap.** A 'snap' occurs when two fractions with the same value (i.e. equivalent fractions) are put down (e.g. $\frac{8}{12}$ and $\frac{18}{27}$ – both equivalent to $\frac{2}{3}$).

This is a difficult activity as you will soon find out if you play with your children. It is not at all obvious, for example, that $\frac{8}{12}$ is equivalent to $\frac{18}{27}$!).

3. Fraction Whist. Deal five cards to each person playing. In turn each player lays down one card. The person who lays down the card of highest value is the winner. As there are twelve equivalent fractions in each suit, if two or more people lay down the same highest value fraction ($\frac{9}{12}$, $\frac{18}{24}$ and $\frac{15}{20}$ for example), the one with the largest numerator and denominator is the winner ($\frac{18}{24}$ in this case) since they need to do more calculation to find the fraction in its lowest terms. The winner keeps the five cards just played as a 'trick'. The players then lay another card and so on. The one with the highest number of tricks is the overall winner for that game and gets to deal in the next game.

4. Matching pairs. The whole pack is laid out on a table in a 6×8 grid, face down. Each player takes it in turn to turn over two cards. If they are equivalent fractions that player keeps the cards as a 'trick'. If they are not they are put back in exactly the same places. The person with the most tricks at the end is the winner. This game not only promotes reinforcement of equivalent fractions, it also encourages memory and concentration skills.

5. Adds to 1. Each player is dealt five cards. He/she looks at the cards and if there are any pairs that add to 1 such as $\frac{1}{2}$ and $\frac{2}{4}$ or $\frac{5}{15}$ and $\frac{14}{21}$, these are put down as 'tricks' immediately in front of the player. The dealer then offers each player in turn a chance to swap one their cards for a new, unseen, card from the top of the pack. If this adds to one with one of their remaining cards, the player immediately puts this pair down in front of them. This continues until one player has put down all his/her cards and is declared the winner. If no more matches can be found the winner is the one who has the most tricks.

Two things need to be noted. Firstly, rotate the dealer one place for each game. Secondly, make sure you are using a pack that contains fractions that do add to one!



Interesting Question:
"What do you have to divide a half by to get two?"

6. Addition and subtraction. I must warn you this is very difficult and should not be attempted until real competence with equivalent fractions has been achieved. Your children may need a lot of help with this, so don't try it if you are not competent with fractions yourself – leave that to the teacher.

On a clean sheet of paper put down two fraction cards chosen from the pack at random such as $\frac{24}{32}$ and $\frac{9}{27}$ and add them together or subtract the smaller from the larger.

$$\boxed{\frac{9}{12}} + \boxed{\frac{10}{30}} = \frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1\frac{1}{12}$$

You can probably see why this is difficult. First we need to reduce the numerators and the denominators by cancelling to their lowest terms (i.e. the basic fractions that will not cancel any more) and then increase them again by lecnacing so that the denominators are the same (12 in this case) before we can proceed with the addition.

As I said earlier, if you are not confident with this, leave it to the teacher and just concentrate on the equivalent fraction side of things.

When your children are proficient with these four fractions ($\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$) and their equivalents, remove the twelve cards representing the simplest fraction and replace them with a harder fraction such as $\frac{3}{5}$ and eleven equivalent fractions. By gradually increasing the difficulty level in this way, it is surprising how competent children can become. Remember at all times, though, to keep it fun and stop before they become bored or stressed – there's always tomorrow!

Fraction Bingo: Make up some **bingo cards** with fractions on each square.

On each card there will be just four unique fractions, but also many of their equivalents. Each card should have a different combination of unique fractions.

The caller calls out fractions slowly, one at a time, and the players cross out the fractions called and all the equivalent fractions to these on their board.

You can play for the first person to get one row across, down or diagonally, or the first person to get all their fractions crossed off.

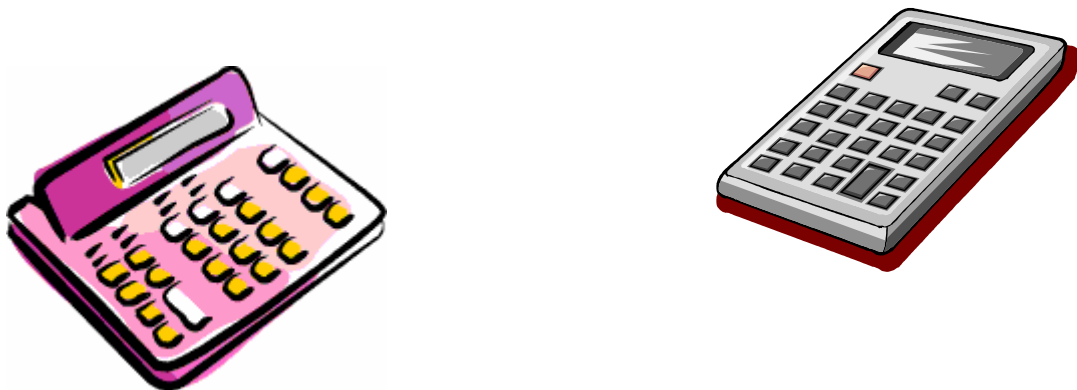
Calculators

There is a great deal of discussion as to whether children should be using calculators and at what age they should begin. Having taught mathematics for many years, I would say the general consensus among teachers of mathematics is that calculators have a very important part to play because

- a)** they give opportunities to explore aspects of mathematics that used to take much longer before calculators were invented. E.g. during a typical lesson, it is now possible to calculate the values for and draw several graphs whereas before there was only time to calculate and draw one graph. This means the teacher and class can concentrate much more on how a graph changes when the numbers in the equation change thus revealing more about the nature of equations
- b)** calculators may be used to check calculations performed manually, thus encouraging quicker feedback on the learning process
- c)** calculators introduce an element of fun into lessons and this encourages children to work harder on the other important aspects of the lesson.

However, the use of calculators should never replace a good knowledge of mental arithmetic because, of course, calculators will not always be available when you need to do a calculation and a good knowledge of mental arithmetic is very useful when studying a whole range of topics such as fractions, square numbers, prime numbers and basic geometry.

There is little point in repeating the activities that teachers will be doing in class as that encourages boredom in lessons, so for the most part I have included fun activities that encourage practice of the use of the calculator that may be introduced at home.



1. Missing numbers in a sequence. Earlier I talked about seeing patterns in house numbers and these provide a good opportunity to practise number sequences ('What is the next house number on this side of the street after this house/after grandma's house?' etc.) Using a calculator, this activity can be extended to more difficult sequences (and, indeed, much more difficult sequences if your children can manage the concepts.)

Here are some examples. **Find the correct level from this set for your child and make up some similar ones.**

What are the next four numbers in these sequences:

1, 4, 7, 10, 13,

4, 11, 18, 25, 32, ...

153, 181, 209, 237, ...

3.5, 6.0, 8.5, 11, ...

18.42, 30.87, 43.32, 55.77 ...

-15, -12, -9, -6, ...

What four numbers come before these:

..., 24, 28, 32, 36

..., 28, 25, 22, 19

..., 198, 235, 272, 309

..., 18.6, 21, 23.4, 25.8

..., -16, -19.5, -23, -26.5

What are the missing numbers?:

18, 24, ..., 36, 42

19.5, 26.8, ..., 41.4, 48.7

16, ..., ..., ..., ..., 41, ..., ..., ..., 61

Enough of this skating lark, can we get on with some series work now, mum?



Once you have established the type of problem that is suitable for your child, it is a simple matter to make up similar ones. You can make it more fun by getting the child to make up some for you to try.

2. Triplets. It is best if you prepare this in advance. Choose a group of five or six numbers such as 19, 23, 57, 38 and 76 and, taking them three at a time, work out the totals or combinations of totals and differences such as:

$$23 + 57 + 76 = 156 \quad \text{or}$$

$$76 + 19 - 38 = 57$$

Then remove the numbers on the left hand side of the expressions leaving just the signs and answers:

$$\underline{\quad} + \underline{\quad} + \underline{\quad} = 156$$

$$\underline{\quad} + \underline{\quad} - \underline{\quad} = 57$$

Give the children the list of numbers you began with and let them find the missing numbers in the expressions. (more time consuming than you might think!)

You can increase the difficulty level of this exercise quite easily by making the list of starting numbers longer, by including larger numbers or decimals. Some of the answers could be negative, e.g.

$$\underline{\quad} + \underline{\quad} - \underline{\quad} = -34$$

Don't forget to allow the children to make up some for you to do – this gives them plenty of calculator experience too!

3. Calculator Nim. Two children (or one child and you) have one calculator between them. You must only use the keys 1 to 9 and the + and =.

Starting with zero in the calculator, each player in turn adds a single digit to the number in the display. The first player to go over 40 (or any other agreed total) is the loser.

Notice that players are not allowed to add 0 or negative numbers or decimal numbers.

As usual, you should always look for ways to vary the game to suit different abilities and ages. With this game, for example, you can change the target number, use subtraction instead of addition, add a two- or three- digit number each time and aim for a much bigger target. The point here is that the children can only use the calculator for adding onto the total. How much they are going to add on has to be worked out in their heads, so **this is a very good combination of calculator and mental work.**

4. Powers. You will remember from your own school days that 3^4 means $3 \times 3 \times 3 \times 3$ and this evaluates to 81. The important point is that 3^4 is not $3 \times 4 = 12$!

Find the value of 6^3 , 4^8 , 3.6^7 , again choosing problems of the correct level for your child and making up similar ones.

What is the value of n in 4^n just before the answer is too big for your calculator? (In other words how many times can 4 be multiplied by itself before the whole answer will no longer fit in the display. For an eight digit calculator the answer is about 13).

Which number in each pair is greater in value?:

3^4 or 4^3 7^5 or 5^7 10^2 or 2^{10} 8^5 or 5^8

Is there a rule? In other words can you tell which is bigger, 12^7 or 7^{12} without working out the answers?

5. Largest number. Children are allowed to use the number keys (or a limited number of the number keys for younger children) and the operation +. Each number key can be used only once, but the + key can be used as many times as desired. (It would be great to also use the \times sign, but as soon as they start multiplying a few numbers together, the answer will soon become greater than can be displayed in the calculator).

What is the largest number that can be made?

Here children can make up sums such as $345 + 982$
or $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$

Less experienced children think that the second sum will produce a greater answer than the first because it contains a greater number of digits, but this shows they need more practice with place value which this exercise gives them, hence its value. Remember to play along too, but let them win sometimes - don't hog all the sweets!

Other things will gradually become apparent to them too, such as that it is better to use numbers with the larger digits in the higher value places in the numbers. For instance $987 + 654$ will give a larger answer than $789 + 456$. This all seems obvious to us, but many children still have to learn this.

What is the very largest number your children can think of?

6. It's not all Zeroes. This game works best with a small group or pair of people. Type a five figure number into a calculator such as 78259.

Pass the calculator to the next person. This person must subtract a number so that the answer ends with a zero. The next person should subtract a number so that the answer ends with two zeroes, the next with three zeroes etc until the answer is zero.

In this example, the first person would subtract 9
the second person would subtract 50
the third person would subtract 200 and so on.

If a person makes a mistake, the incorrect answer stands and the next person has to deal with this. For example, if the second person subtracted 5 instead of 50, the calculator would now show 78245 and the third person would have to subtract 245 so that the answer ends with three zeroes.

This is a brilliant game to practice place value (which is the fundamental keystone on which the modern number system is built), as many children will want to subtract just a single digit (as in the error described above) instead of a digit followed by a number of zeroes.

This game can be extended by trying to make the final answer 11111 or 22222 instead of zero (or 54321 come to that – just thought of this idea!). Of course, you can always use decimals too.

It is also good to play the same game on paper, where each child has to write down the number he/she is going to subtract and once written it cannot be changed. The answer also has to be worked out on paper.

You can change the operation to addition and try to make 100 000 or 1 000 000.

7. Palindromes. Write down a number. Write down the same number underneath, but reversed. Add the two numbers.

a) is the answer palindromic? (i.e. is it the same backwards as forwards?)

b) is it divisible by 11.

E.g.	Write down	3516
	Reverse	<u>6153</u>
	Add	9669

As it happens, 9669 is both palindromic and divisible by 11.

You should find that sometimes the answer is palindromic and sometimes it is divisible by 11. If not both, you can repeat the process with the answer and keep doing this until the answer is both palindromic and divisible by 11.

8. Don't you dare! A difficult game for two players who must have a good understanding of decimals (or the whole thing is a bit pointless!)

Each player thinks of a number and enters it into his/her calculator. Both players are allowed to see each other's calculators at all times.

The one with the higher number subtracts any number he/she wants from his total and the one with the lower number adds any number to his/hers.

The idea is not to cross the number in the other person's display (i.e. the one who is subtracting must not go below the other person's number and the one adding must not go above the other person's display).

The reason this gets tricky is that as the two numbers become close, it is necessary to start subtracting or adding decimals and smaller and smaller decimals at that!

The first to cross their partner's number is the loser.

The slight problem with this game is that smart children spot that if you subtract or add a very small number such as 0.0000001 , the game goes on almost for ever and no-one will lose. If this happens, it will be necessary to impose an absolute lowest amount (such as 1 or 0.5 or 0.05 that can be added or subtracted).

9. The Magic of 101! Take a two digit number such as 23 and multiply it by 101 on the calculator. Try this several times and see if you can find a pattern.

Now try with numbers of the form $aabb$ where $a + b$ is not larger than 9 such as 3366 and multiply these by 101 . Can you see any patterns?

Extend this idea to multiplying by 1001 etc.

10. More Palindromes. Find a palindromic number (a number that reads the same backwards as forwards) that is a multiple of 3 .

After that try to find palindromic numbers that are multiples of $4, 5, 6, 7, 8, 9, 11, 12$ or 13 (not all at the same time, of course)

Find palindromic numbers that are
a) a square number b) a triangle number c) a prime number

(Not sure what these are? Download the dictionary from www.mathsgogogo.co.uk)

11. Attractors. This is an activity for more intelligent or experienced children as it is rather abstract in design. You will need a calculator with a square root ($\sqrt{}$) key.

Take a number such as 45 and find its square root. Add another fixed number such as 9 and then keep repeating the same process:

Take the square root of the answer and add nine.

Take the square root of the answer and add nine.....

You will find the answer homes in on a fixed number (in this case 12.54138126...)

Try different pairs of numbers. The number that the calculation homes in on is called an 'attractor'.

The great thing about this exercise is that it doesn't matter if a mistake is made at some point – eventually the series will be attracted back to the same number (try it and see!).

Well, I could go on for a very long time, but I hope that this has given you a flavour of the possibilities.

It is surprisingly easy to help your children with mathematics at home, both in and out of the house.

Look for the many opportunities that are around you and remember that most parents don't do this, so anything you are prepared to do with your children will give them a good start compared to their peers. Children love learning, so why not get them ahead while they are keen?

Wishing you the very best of success.

Alan Young

www.mathsgogogo.co.uk